Getting Schooled

Hello and welcome to this edition of Getting Schooled. In keeping with our theme of interventions in School Psychology, Dr. Michelle Drefs, Ms. Katherine Winters and myself have written an article on math interventions. We have chosen to focus on the importance of number sense as a foundational process, highlighting its crucial importance in the development of mathematical understanding and skills.

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Math Interventions that Make Sense

If the lay public and educators were asked what two academic areas are most important for children to learn to be successful in school, there is little doubt a majority would say reading and mathematics…in that order (Clarke, Baker, & Chard, 2008, p. 453).

Mathematical ability has been shown to have a significant impact on employment opportunities, economic status, and physical and mental well-being (Bynner & Parsons, 2005). Not surprisingly then, math learning difficulties are among the primary reasons students are referred to school psychologists. As such, it is essential that school psychologists understand math learning difficulties and are able to select effective instructional techniques and interventions to support students who struggle with mathematics. The need for such expertise is likely to further increase as schools and educators experiencing success in the delivery of multi-tiered models of instructional supports for reading begin to shift their focus and seek out similar multi-tiered models of support for early mathematics (Clarke, Doabler, & Nelson, 2014).

As outlined in the DSM-5, students who meet criteria for a diagnosis of Specific Learning Disorder with impairment in mathematics experience remarkable difficulty with one or more of the following four subskills: number sense, memorization of arithmetic facts, accurate or fluent calculation, and/or accurate math reasoning (American Psychiatric Association, 2013). Discussing instructional techniques and interventions for each of the four subskills is beyond the scope of this brief article. As such, we decided to focus on number sense – largely based on significant and recent advancements in our understanding of what number sense is and how best to support the development of this foundational subskill. We begin by providing a brief history of the research on the importance of number sense, followed by an overview of key areas to consider in selecting effective interventions for students with underdeveloped number sense.

Research on the Importance of Number Sense

In many respects, number sense is the "new kid on the mathematics block." This is evidenced, for example, in the notable absence of number sense from some of the earlier and more prominent models of mathematical learning disabilities (for example, see Geary, 2004).

Although it is almost twenty years since Gersten and Chard (1999) first proposed that the relationship between number sense and mathematics is analogous to the importance of phonemic awareness for reading, it is only within the last decade or so that interest in number sense among practitioners has peaked. Fuelling this rise is compelling research from the fields of mathematics education, cognitive psychology, and neuroscience supporting number sense as a critical determinant of both early and more advanced mathematics performance. In general, we know that most children have a fundamental sense of quantity and a basic understanding of numbers well before they enter school around 5 years of age (Friso-van den Bos et al., 2015; Geary, 2013). Unfortunately, children who lack or fail to obtain a solid understanding of number in the early grades are unlikely to catch up and are at-risk of performing significantly below their peers in mathematics throughout their schooling (Griffin, 2004). Furthermore, there is evidence to suggest that early math skills are even stronger predictors of later academic achievement than reading skills, attention, or behaviour (Duncan et al., 2007).

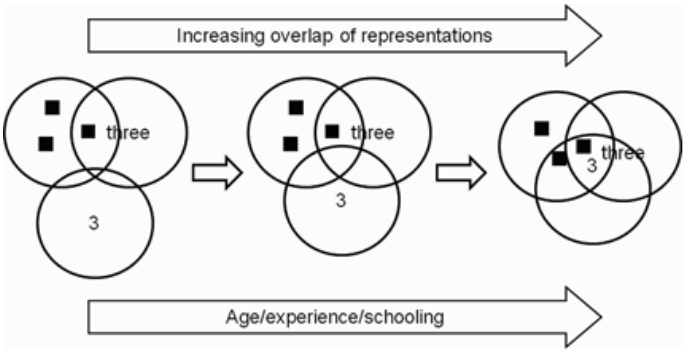
Interventions for Number Sense: Basic Considerations

While we presently know a great deal about number sense, it is important to keep in mind that considerably less time and study has been given to this area compared to what has been given to deficits in other subskills associated with math learning difficulties (e.g., fluent retrieval of basic arithmetic facts) or to other academic domains (e.g., reading). Accordingly, we are only in the initial stages of understanding how best to intervene in this area.

To fully appreciate the recent literature related to effective number sense interventions, it is helpful to begin with a brief description of what constitutes number sense. This is somewhat challenging however, due to inconsistencies in how number sense is defined and operationalized, both within and across disciplines. In the broadest sense, number sense refers to a "fluidity and flexibility with numbers" (Gersten & Chard, 1999, pg. 19). More specifically, number sense is a developmental process that involves both the innate ability to process discrete and continuous nonsymbolic quantities (e.g., judging which of two collections is larger), as well as basic symbolic number competence (i.e., recognizing Arabic numerals and understanding how numbers relate to one another), acquired through experience and formal schooling (e.g., oral counting; number identification, sequencing, and magnitude comparison; mental calculation). Current research investigating the relative importance of nonsymbolic versus symbolic numerical skills to mathematical outcomes at different developmental periods suggests that while symbolic number skills matter more as children age (Merkley & Ansari, 2016), nonsymbolic numerical skills are associated with mathematics performance in younger children (prior to age 7) and those with persistent math learning difficulties (e.g., Brankaer, Ghesquière, & De Smedt, 2014).

From our perspective, central to a school psychologist's ability to effectively support students with math learning difficulties is viewing number sense as the process of building an increasingly sophisticated understanding of the relationships between numbers and integrating the various representations of number (e.g., perceptual forms, number symbols) – as opposed to simply a collection of skills (e.g., counting to 20, comparing magnitudes). To illustrate this approach to understanding number sense, we present Kucian and Kauffman's (2009) developmental model (Figure 1) depicting how, with age and formal instruction, the overlap between various representations of quantity increases and eventually creates a cohesive abstract

representation of number. For instance, a young child may use “three” as a verbal tag within a counting sequence (“1, 2, 3”) without associating it with the quantity it represents. For the student with a well-developed number sense, however, the various number representations are inextricably fused such that “3” is additionally understood as “1” more than “2” and half of “6.” It is this eventual integration of previously isolated skills into a coherent whole that is the quintessential characteristic of number sense and allows children to develop the fluidity and flexibility with numbers that is necessary to become mathematically proficient. Accordingly,



interventions targeting number sense need to focus on this integration of representations.

Figure 1. Developmental model of number representation showing increased “overlap of representations across different number notations”. Reprinted from “A developmental model of number representation,” by K. Kucian and L. Kaufmann, 2009, *Behavioral and Brain Sciences, 32*, p. 341. Copyright [2009] by Cambridge University Press.

Given the apparent benefit of being able to fluidly and flexibly interpret various numerical representations and use them to think mathematically, there are several interventions designed to allow students to learn and practice their understanding of numbers. For example, the Number Sense Intervention (NSI; Dyson, Jordan, Beliakoff, & Hassinger-Das, 2015; Dyson, Jordan, & Glutting, 2013) is an evidence-based math program that provides students with instruction in three fundamental whole-number concepts that are important for learning mathematics: number, number relations, and number operations. Students are taught skills such as subitizing, or "seeing" and labeling quantities represented as groups of counters. Through the use of cardinality charts and number lists, instructors also relate these representations to the symbolic numerals and demonstrate the "plus one" principal by showing students that any number plus one more equals the next number in a series and, similarly, that any number less one equals the previous number. These strategies and tools are used to introduce the cardinality principal, or the idea that the last number counted is the total number of items in a set. It also teaches strategies such as "counting on," a precursor to basic addition that encourages young students to work from a specific number set by beginning counting from that number to get to a larger number set, instead of starting from 1 to count the entire set of numbers. For example, if a student knows a specific subset of numbers (e.g., the number of fingers on one hand), he or she can hold this quantity in mind and begin counting from there to get to a greater total (Fuchs et al., 2013).
The NSI program also teaches part-whole relationships by introducing the concept of number "partners" (Fuson, Grandau, & Sugiyama, 2001). This concept involves understanding that numbers 2 and greater can be made up of "partners," or pairs of smaller numbers, that “work

together" to create the larger number (e.g., 4 can be made up of 1+3, 2+2, or 3+1). The "partners" are used in flexible ways to solve problems, showing the relationship between the entire set of items, its partner numbers, and the numerals that are associated with it. This begins to introduce basic addition and subtraction operations, using all of the representations related to number sense.

There is evidence to suggest that when low achieving kindergarten students participate in programs such as the NSI, their number sense and math achievement improve (Dyson, Jordan, & Glutting, 2013). The NSI was found to be particularly effective when it was paired with number fact practice (i.e., providing quick answers to addition and subtraction combinations) with long-term gains on measures of number sense, arithmetic fluency, and mathematics achievement (Dyson et al., 2015).

Summary

There are currently a wide number of both commercial and open access interventions available, that target the development of number sense. In addition to attending to best practice guidelines for selecting evidence-based interventions, school psychologists need to consider the potential merits of number sense interventions based on the extent to which they promote the integration of multiple representations of number. Relatedly, school psychologists should also ensure their assessment toolkits include number sense screeners and measures. It is possible, for example, that a student who appears to have poorly developed mathematical reasoning skills may, in fact, be lacking more foundational number sense competencies.

**A copy of the full reference list is available upon request.*

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